Name: ______________________ Section: ______________________

Taking Derivatives Efficiently

1. Know the basic derivative rules backwards and forward.
   (a) Polynomials and exponential functions
   (b) The product and quotient rules
   (c) Trigonometric functions and $\ln(x)$
   (d) $f''(x)$ is the rate of change of $f'(x)$. You find f'' by taking two derivatives of f. Similarly, $f^{(n+1)}$ is the rate of change (i.e. slope) of $f^{(n)}$.
2. Antiderivatives: Know the derivative rules backwards
   (a) Be able to find general antiderivatives
   (b) Be able to find the constant C when you are given additional information. For example, if you know $f'(x)$ and if you know $f(0) = 1$, you can find $f(x)$ exactly.
3. The Chain Rule can be written three ways (each has its own use).
   (a) $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$
   (b) $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, where u stands for the inside function.
   (c) $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot \frac{d}{dx}[g(x)]$
4. Implicit Differentiation

   Remember that in these problems, y is a "hidden" function of x.
   There are two steps: (1) take the derivative of both sides and (2) solve for y'
5. Differentiation and Logarithms
   (a) When taking the derivative of a function containing logarithms, try using the 3 laws of logarithms to simplify first.
   (b) Introduce logarithms when computing certain complicated derivatives

Concrete Applications of Derivatives

1. Exponential growth and decay
   (a) Know the general formula $P(t) = P_0 e^{kt}$
   (b) Be able to find the constants.
   (c) Be able to find the time needed for the population to double, or to reach a given size.
2. Related Rates. *See the Related Rates Worksheet* for the key examples to know.
3. Optimization Problems. *See the Optimization Worksheet* for the key examples to know.

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Abstract Applications of Derivatives

1. Linear approximations via derivatives.
   (a) The tangent line to $f(x)$ at a is a line that approximates $f(x)$ near a.
   (b) Know the formula for $L(x)$.
   (c) For numbers x near a, $f(x) \approx L(x)$.
2. Given the graph of a function, be able to identify
   (a) intervals where f is increasing/decreasing and concave up/down
   (b) the x values of local maxima/minima, absolute maxima/minima, and points of inflection
3. The graphical meaning of derivatives
   (a) The first derivative describes slope (increasing/decreasing)
   (b) The second derivative describes concavity (concave up/down)
4. Local Maxima and Minima
   (a) Critical points: when *might* f have a local max/min?
   (b) The first derivative test. Check for extrema by thinking about the function's slope.
   (c) The second derivative test. Check for extrema by thinking about concavity.
5. Absolute Maxima and Minima
   (a) Every continuous function f restricted to a closed interval $[a, b]$
   has an absolute max *and* an absolute min in $[a, b]$.
   (b) To find the max/min on a closed interval, just check the value of f on the critical points and the endpoints.
   (c) There is also a first derivative test for absolute max/min.
6. Using *L'Hospital's Rule* to find limits
   (a) Know L'Hospital's rule, and know when you can *and cannot* apply it.
   (b) Indeterminate quotients are type $\frac{0}{0}$ and $\frac{\infty}{\infty}$
   (c) Indeterminate products are type $\infty \cdot 0$.
   You must turn these into quotients before you can apply L'Hospital's rule.
   (d) Indeterminate powers have type 1^∞, 0^∞, and ∞^0.
   You must introduce logs, and turn them into quotients.